Neutrino dispersion in magnetized plasma

N. V. Mikheev,* E. N. Narynskaya†

Yaroslavl State University

150000 Russian Federation, Yaroslavl, Sovietskaya 14.

Abstract

The neutrino dispersion in the charge symmetric magnetized plasma is investigated. We have studied the plasma contribution into the additional energy of neutrino and obtained the simple expression for it. We consider in detail the neutrino self-energy under physical conditions of weak field, moderate field and strong field limits. It is shown that our result for neutrino dispersion in moderate magnetic field differ substantially from the previous one in the literature.

1 Introduction.

The investigations of neutrino physics in an active medium are the subject of great interest today. The one of the topical problem in current researches is the influence of an external magnetic field and plasma on the neutrino dispersion relation.

It this paper we analyse the neutrino dispersion properties in an active medium consisting of magnetic field and plasma. The investigations of such type are based on calculation of neutrino self-energy operator $\Sigma(p)$. This operator can be defined in term of the invariant amplitude of the neutrino transition $\nu \to \nu$, by the relation:

$$M(\nu \to \nu) = - \bar{\nu}(p)\, \Sigma(p)\, \nu(p), \tag{1}$$

where p^μ is the neutrino four-momentum.

Using the expression (1), one obtains for the additional neutrino energy in magnetized plasma [1]:

$$\Delta E = \frac{1}{4E}\, Sp\left\{ ((p\gamma) + m_\nu)\,(1 - (s\gamma)\,\gamma_5)\,\Sigma(p) \right\}, \tag{2}$$

where E is the neutrino energy in vacuum, m_μ is the neutrino mass, s^μ is the neutrino spin four-vector, γ^α are the Dirac matrices in the standard presentation. The Lorentz indexes of four-vectors and tensors within parenthesis are contracted consecutively, for example, $(p\gamma) = p^\mu \gamma_\mu$.

There are several parametrization for general structure of the operator $\Sigma(p)$. In the presence of homogeneous magnetized medium the operator $\Sigma(p)$ contains three independent structures. In this case it is convenient to express $\Sigma(p)$ as

$$\Sigma(p) = \left\{ a\,(p\gamma) + b\,(u\gamma) + c\,(p\tilde{\varphi}\gamma) \right\}\, L. \tag{3}$$

Here a, b, c are the numerical coefficients, u^μ is the four-vector of medium velocity, $\tilde{\varphi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}/B$ is the dimensionless dual tensor of the magnetic field, B is the absolute value of the magnetic field strength, $L = (1 - \gamma_5)/2$ is the left-handed projection operator.

---

*e-mail: mikheev@uniyar.ac.ru

†e-mail: elenan@uniyar.ac.ru

[1] We use natural units in which $c = \hbar = 1$.

On this parametrization the coefficients a, b, c have a simple physical interpretation. Really, performing calculation of the trace in eq.(3) with $\Sigma(p)$ in the form (2), one can obtained for the neutrino self-energy

$$\Delta E = b\, \frac{(1 - (\vec{v}\vec{\xi}))}{2} - c\, \frac{m_\nu}{2E}\, (p\tilde{\varphi}s). \tag{4}$$

where $\vec{v}$ is the neutrino velocity, $\vec{\xi}$ is the twice vector of average spin of neutrino.

It is seen that the additional energy in magnetized plasma for massless left-handed neutrino in fact depends on the parameter b only,

$$\Delta E = b.$$

In the case of massive neutrino the second term in (4) corresponds to the additional energy caused by a neutrino magnetic moment μ_ν,

$$\Delta E_{\mu_\nu} = -\frac{\mu_\nu\, B}{E}\, (p\tilde{\varphi}s), \qquad \mu_\nu = \frac{c\, m_\nu}{2B}.$$

So, one can speak that parameter c determines the additional neutrino magnetic moment in the magnetized plasma.

The modification of the neutrino dispersion relation in a magnetized plasma have a long history, see for example [1]-[5]. In particular, the dispersion relation of neutrino [2] in charge symmetric plasma under physical conditions

$$m_W^2 \gg T^2,\; eB \gg m_e^2, \qquad eB \le T^2 \tag{5}$$

is well known

$$\frac{\Delta E}{|\vec{p}|} = \frac{\sqrt{2}\, G_F}{3} \left[-\frac{7\,\pi^2\, T^4}{15} \left(\frac{1}{m_Z^2} + \frac{2}{m_W^2} \right) + \frac{T^2\, eB}{m_W^2}\, \cos\phi + \frac{(eB)^2}{2\,\pi^2\, m_W^2}\, \ln\left(\frac{T^2}{m_e^2} \right) \sin^2\phi \right]. \tag{6}$$

Here $\vec{p}$ is the neutrino momentum, ϕ is the angle between magnetic field direction and vector $\vec{p}$, T is the plasma temperature.

The first term in (6) is the pure plasma contribution [1], while the second [2] and third [3] terms in brackets are caused by the common influence of the plasma and magnetic field. As one can see the term of the second order of the field contains the infrared divergence in the massless electron limit. However, there is a good reason to think that this large logarithmic factor $\ln(T^2/m_e^2)$ can not arise under physical conditions (5), when the electron mass is the smallest parameter of the task. Actually, under conditions (5) the contribution into neutrino energy is determined by electrons and positrons on exited Landau levels with energy $\omega_n = \sqrt{k_3^2 + 2eBn + m_e^2}$. One can see that in this case the electron mass squared can be neglected in comparison to magnetic field. By this means, the availability of the logarithmic factor $\ln(T^2/m_e^2)$ in the (6) has come into question, so independent investigations of the neutrino dispersion in magnetized medium it is required.

Further we study the additional neutrino energy for massless neutrino in the charge symmetric plasma with the presence of the magnetic field of arbitrary strength. As a special case we consider the weak field, moderate field and strong field limits in more details.
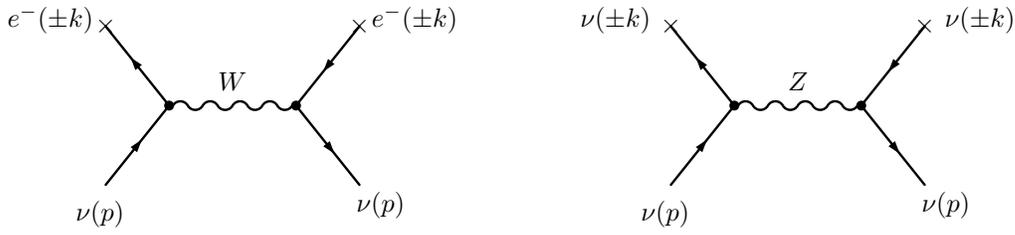
---

[2] we consider the electron neutrino

Figure 1: Feynman diagrams for the plasma contribution into the neutrino self-energy operator.

2 The plasma contribution into neutrino self-energy operator $\Sigma(p)$.

In the case of the charge symmetric plasma the contribution into amplitude of neutrino transition $\nu \to \nu$ in magnetized plasma derives from processes, shown in fig.1. It should be noted that in the local limit the neutrino self-energy operator $\Sigma(p)$ is zero [1]. Thus, there is a need to take into account the momentum dependence of W,Z - boson propagators. The process of neutrino transition via Z boson does not sensitive to influence of external magnetic field. Thus, the contribution into operator $\Sigma(p)$ from this process can be extracted from paper [1].

The amplitude of neutrino forward scattering on all plasma electrons and positrons can be immediately obtained from the Lagrangian of νe interaction with W-boson

$$L = \frac{g}{2\sqrt{2}} \left(\bar{e}\,\gamma_\alpha\,(1-\gamma_5)\,\nu \right) W_\alpha + \frac{g}{2\sqrt{2}} \left(\bar{\nu}_e\,\gamma_\alpha\,(1-\gamma_5)\,e \right) W_\alpha^*. \tag{7}$$

Omitting the details of calculations, the general expression for the neutrino self-energy caused by the neutrino forward scattering on plasma electrons can be written as

$$\frac{\Delta E}{|\vec{p}|} = -\frac{2\sqrt{2}\,G_F\,eB}{\pi^2\,m_W^2} \int\limits_{-\infty}^{+\infty} \frac{dk_3\,f(\omega_n)}{\omega_n} \times \tag{8}$$

$$\times \left(\sideset{}{'}\sum_{n=0} (\omega_n^2 + eBn + \cos^2\phi\,(k_3^2 - eBn)) - \frac{\delta_{n0}}{2}\cos\phi\,(k_3^2 + \omega_n^2) \right).$$

Here $\omega_n = \sqrt{k_3^2 + 2eBn + m_e^2}$ and k_3 are the electron energy and z-component [3] of electron momentum correspondingly, n is the Landau level number, ϕ s the angle between magnetic field direction and neutrino momentum $\vec{p}$, $f(\omega_n)$ is the electron distribution function, $f(\omega_n) = [exp(\omega_n/T) - 1]^{-1}$, the sum is defined as

$$\sideset{}{'}\sum_{n=0} F(n) = \frac{1}{2}\,F(n=0) + \sum_{n=1}^{\infty} F(n).$$

The integral and sum in eq. (8) can be calculated under some physical conditions:

- the limit of weak magnetic field, when the magnetic field strength is the smallest physical parameter

$$T^2 \gg m_e^2 \gg eB. \tag{9}$$

---

[3] the magnetic field is directed along the z-axis

The result of calculations for the plasma contribution into neutrino self-energy in this limit is

$$\frac{\Delta E}{|\vec{p}|} = \frac{\sqrt{2}\,G_F}{3m_W^2}\left[-\frac{7\,\pi^2\,T^4}{15}\left(2+\frac{m_W^2}{m_Z^2}\right)+T^2\,eB\,\cos\phi+\right. \tag{10}$$

$$\left.+\frac{(eB)^2}{2\,\pi^2}\left\{\sin^2\phi\left(\ln\left(\frac{T^2}{m_e^2}\right)+0,635\right)-1\right\}\right].$$

As one can see the expression (10) contains the logarithmic factor with m_e, but under conditions considered (9) the electron mass is not the smallest parameter of task. So the electron mass can not be tend to zero at nonzero magnetic field B.

• the moderate magnetic field, when the magnetic field being relatively weak in comparison with plasma temperature, is simultaneously strong enough on the scale of electron mass squared

$$T^2 \gg eB \gg m_e^2. \tag{11}$$

Under conditions (11) plasma electrons and positrons occupy highest Landau levels. In this case for neutrino energy one can obtained

$$\frac{\Delta E}{|\vec{p}|} = \frac{\sqrt{2}\,G_F}{3m_W^2}\left[-\frac{7\,\pi^2\,T^4}{15}\left(2+\frac{m_W^2}{m_Z^2}\right)+T^2\,eB\,\cos\phi+\right. \tag{12}$$

$$\left.+\frac{(eB)^2}{2\,\pi^2}\left\{\sin^2\phi\left(\ln\left(\frac{T^2}{eB}\right)+2,93\right)-1\right\}\right].$$

Our calculations show, that under conditions (11) the additional neutrino energy does not contain the infrared divergence in the limit $m_e \to 0$ in contrast [3].

• the strong magnetic field limit, when from two components of active medium the field component dominates

$$eB \gg T^2 \gg m_e^2. \tag{13}$$

Under conditions (11) the most part of plasma electrons and positrons occupy the ground Landau level. The result for the additional neutrino energy in this limit has the form:

$$\frac{\Delta E}{|\vec{p}|} = -\frac{\sqrt{2}\,G_F}{3m_W^2}\left[\frac{7\,\pi^2\,T^4\,m_W^2}{15\,m_Z^2}+\frac{T^2\,eB}{2}\,(1-\cos\phi)^2+\right. \tag{14}$$

$$\left.+3\,(eB)^2\left(\frac{2}{\pi}\right)^{3/2}\left(\frac{T^2}{2eB}\right)^{1/4}(3-\cos^2\phi)\,e^{-\sqrt{2eB/T}}\right].$$

Here the second term corresponds to the contributions of ground Landau level, while the third term is caused by the first Landau level.

3 Conclusion.

We have studied the neutrino dispersion in the charge symmetric plasma with the presence of a constant magnetic field. The most general expression in simple analytic form for the plasma contribution into the neutrino self-energy was obtained. In particular, we have considered the physical conditions, corresponding to the weak field and moderate field, when plasma electrons and positrons occupy the excited Landau levels. The strong magnetic field limit, when plasma electrons and positrons mainly occupy the lowest Landau level, is investigated also.

It is shown that additional neutrino energy in the limit of moderate field, $T^2 \gg eB \gg m_e^2$, does not coincide with previous result in paper [3] under the same physical conditions and does not contain the infrared divergence in the limit $m_e \to 0$ in contrast to [3].

References

[1] D.Notzold, G.Raffelt, Nucl.Phys.B **307**, 924, (1988).

[2] P. Elmfors, D. Grasso and G.Raffelt, Nucl.Phys.B **479**, 3, (1996).

[3] A. Erdas, G.W.Kim, T.H.Lee, Phys. Rev. D **58**, 085016, (1998).

[4] E. Elizalde, E.J.Ferrer, V. de la Incera, Phys. Rev. D **70**, 043012, (2004).

[5] A.V. Kuznetsov, N.V.Mikheev, G.G.Raffelt, L.A.Vassilevskaya, Phys. Rev. D **73**, 023001, (2006).