Influence of the photon - neutrino processes on magnetar cooling

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Abstract

The photon-neutrino processes $\gamma e^{\pm} \to e^{\pm}\nu\bar{\nu}$, $\gamma \to \nu\bar{\nu}$ and $\gamma\gamma \to \nu\bar{\nu}$ are investigated in the presence of a strongly magnetized and dense electron-positron plasma. The amplitudes of the reactions $\gamma e^{\pm} \to e^{\pm}\nu\bar{\nu}$ and $\gamma\gamma \to \nu\bar{\nu}$ are obtained. In the case of a cold degenerate plasma contributions of the considering processes to neutrino emissivity are calculated. It is shown that contribution of the process $\gamma\gamma \to \nu\bar{\nu}$ to neutrino emissivity is supressed in comparision with the contributions of the processes $\gamma e^{\pm} \to e^{\pm}\nu\bar{\nu}$ and $\gamma \to \nu\bar{\nu}$. The constraint on the magnetic field strength in the magnetar outer crust is obtained.

1 Introduction

Magnetars are highly interesting objects in the our Universe. Recent observations [1–4] give ground to believe that some astrophysical objects (SGR and AXP) are magnetars, a distinct class of isolated neutron stars with magnetic field strength of $B \sim 10^{14} - 10^{16}$ G [5–7], i.e. $B \gg B_e$, where $B_e = m^2/e \simeq 4.41 \times 10^{13}$ G [1] is the critical magnetic field. The spectra analysis of these objects is also providing evidence for the presence of electron-positron plasma in magnetar environment. In addition, the very density matter is attended in the inner layers both the ordinary neutron stars and the magnetars [8].

The understanding of the important role of quantum processes in the magnetar dynamic is the extra stimulus of progress in the astroparticle physics. It is especially important to investigate the influence of external field on the quantum processes where only electrically neutral particles in the initial and the final states are presented, such as neutrinos and photons. The effect of an external field on such processes is associated with two factors. First, charged fermions are sensitive to a magnetic field, a major part being played here by the electron, since this particle has the maximum specific charge. Second, a strong magnetic field has a pronounced effect on the dispersion properties of photons and, hence, on the kinematic of processes with its participation.

Although the matter in neutron stars is very dense ($\rho \lesssim 10^{11}$ g/cm^3 in the outer crust), it is fully transparent for neutrino. Therefore, it is important to study whole variety of neutrino reactions in the young magnetar in order to analyze the cooling. We will consider the following neutrino reactions in a magnetar crust.

- Pair annihilation process, $e^+e^- \to \nu\bar{\nu}$ and synchrotron mechanism, $e \to e\nu\bar{\nu}$ are negligible in strongly magnetized, degenerate plasma [9].

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[1] We use natural units $c = \hbar = k = 1$, m is the electron mass, $e > 0$ is the elementary charge.

- Photoneutrino process, $e\gamma \to e\nu\bar{\nu}$. This process was studied by N. Itoh et al. [10] in the plasma without magnetic field and V. Skobelev [11] in the limit of nonrelativistic degenerate plasma.

- Photon conversion process, $\gamma \to \nu\bar{\nu}$. This process was studied in the two limits weak magnetic field [12] and in strong magnetic field without plasma [13].

- Two photon annihilation, $\gamma\gamma \to \nu\bar{\nu}$. As it was shown in the recent paper [14] the amplitude of this process in strongly magnetized, degenerate plasma will gain by factor eB.

In the present work the process of neutrino cooling is investigated in the presence of strong magnetic field and electron plasma, when the magnetic field strength B is the maximal physical parameter, namely $\sqrt{eB} \gg T,\, \mu\, \omega,\, E$. Here T is the plasma temperature, μ is the chemical potential, ω and E is the initial photon and electron energies. In this case almost all electrons and positrons in plasma are on the ground Landau level. The more accurate relation for magnetic field and plasma parameters in this case can be written in the following form

$$\frac{B^2}{8\pi} \gg \frac{\pi^2 n_{e^-}^2}{eB} + \frac{eBT^2}{12}, \tag{1}$$

where n_{e^-} are electron number densities. In the outer crust of neutron stars the number of electron density can be estimated as

$$n_{e^-} \simeq \frac{m^3}{2\pi^2}\,\frac{\rho_6\, Z}{A}, \qquad \rho_6 = \frac{\rho}{10^6 \mathrm{g/cm}^3}\,.$$

For the typically parameters in the outer crust ($Z = 26$, $A = 56$ and $T < m$ [9]), the condition (1) can be performed up to the density $\rho \sim 10^{10}$ g/cm^3 for the magnetic field strength $B \gtrsim 10^{15}$ G.

Thus, we will investigate the magnetar cooling via neutrino emissivity (energy carried out by neutrinos from unit volume per unit time) with taking into account of the photon dispersion in strong magnetic field and plasma.

2 Photon dispersion in the magnetized medium

The propagation of the electromagnetic radiation in any active medium is convenient to describe in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with eigenvectors and eigenvalues of polarization operator correspondingly. In the case of strongly magnetized plasma in the one loop approximation the eigenvalues of the polarization operator can be derived from the previously obtained results [15–17]:

$$\mathcal{P}^{(1)}(q) \simeq -\frac{\alpha}{6\pi}\left[q_\perp^2 + \sqrt{q_\perp^4 + \frac{(6N\omega)^2 q^2}{q_\parallel^2}}\right] - q^2\,\Lambda(B), \tag{2}$$

$$\mathcal{P}^{(2)}(q) \simeq -\frac{2eB\alpha}{\pi}\left[H\left(\frac{q_\parallel^2}{4m^2}\right) + \mathcal{J}(q_\parallel)\right] - q^2\,\Lambda(B), \tag{3}$$

$$\mathcal{P}^{(3)}(q) \simeq -\frac{\alpha}{6\pi}\left[q_\perp^2 - \sqrt{q_\perp^4 + \frac{(6N\omega)^2 q^2}{q_\parallel^2}}\right] - q^2\,\Lambda(B), \tag{4}$$

where

$$\Lambda(B) = \frac{\alpha}{3\pi}\left[1.792 - \ln(B/B_e)\right], \quad N = \int\limits_{-\infty}^{+\infty} dp_z\, \left[f_-(E) - f_+(E)\right],$$

$$\mathcal{J}(q_\parallel) = 2q_\parallel^2 m^2 \int \frac{dp_z}{E} \frac{f_-(E) + f_+(E)}{(q_\parallel^2)^2 - 4(pq)_\parallel^2}\,, \qquad E = \sqrt{p_z^2 + m^2},$$

$f_\pm(E) = [e^{(E \pm \mu)/T} + 1]^{-1}$ are the electron (positron) distribution functions,

$$H(z) = \frac{1}{\sqrt{z(1-z)}} \arctan \sqrt{\frac{z}{1-z}} - 1, \quad 0 \leqslant z \leqslant 1, \tag{5}$$

$$H(z) = -\frac{1}{2\sqrt{z(z-1)}} \ln \frac{\sqrt{z} + \sqrt{z-1}}{\sqrt{z} - \sqrt{z-1}} - 1 + \frac{i\pi}{2\sqrt{z(z-1)}}, \qquad z > 1. \tag{6}$$

In the case of strongly degenerate plasma ($T \ll \mu,\, m$) one can obtained the analytical expressions for $\mathcal{J}(q_\parallel)$ and N integrals:

$$\mathcal{J}(q_\parallel) = -\frac{1}{2\sqrt{z(1-z)}} \Bigg(\arctan \left[\frac{v_F - v_\phi + z v_F (v_\phi^2 - 1)}{(v_\phi^2 - 1)\sqrt{z(1-z)}} \right] + \tag{7}$$

$$+ \arctan \left[\frac{v_F + v_\phi + z v_F (v_\phi^2 - 1)}{(v_\phi^2 - 1)\sqrt{z(1-z)}} \right] \Bigg), \qquad 0 \leqslant z \leqslant 1,$$

$$\mathcal{J}(q_\parallel) = -\frac{1}{4\sqrt{z(z-1)}} \Bigg(\ln \left[\frac{v_F - v_\phi + (v_\phi^2 - 1)(z v_F - \sqrt{z(z-1)}\,)}{v_F - v_\phi + (v_\phi^2 - 1)(z v_F + \sqrt{z(z-1)}\,)} \right] + \tag{8}$$

$$+ \ln \left[\frac{v_F + v_\phi + (v_\phi^2 - 1)(z v_F - \sqrt{z(z-1)}\,)}{v_F + v_\phi + (v_\phi^2 - 1)(z v_F + \sqrt{z(z-1)}\,)} \right] \Bigg) - \frac{i\pi\theta(v_F|v_\phi| - 1)}{2\sqrt{z(z-1)}}\,, \qquad z > 1,$$

$$z = \frac{q_\parallel^2}{4m^2}\,, \quad v_F = \frac{\sqrt{\mu^2 - m^2}}{\mu}, \quad v_\phi = \frac{\omega}{q_z}, \quad N = 2p_F = 2\sqrt{\mu^2 - m^2}.$$

Here the four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\, 2\}$-subspace and the Minkowski $\{0,\, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along z (third) axis; $(ab)_\perp = (a\Lambda b) = a_\alpha \Lambda_{\alpha\beta} b_\beta$, $(ab)_\parallel = (a\widetilde{\Lambda} b) = a_\alpha \widetilde{\Lambda}_{\alpha\beta} b_\beta$, where the tensors $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta}$, $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta}$, with equation $\widetilde{\Lambda}_{\alpha\beta} - \Lambda_{\alpha\beta} = g_{\alpha\beta} = diag(1, -1, -1, -1)$ are introduced. $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

The dispersion properties of the normal modes could be defined from the dispersion equations

$$q^2 - \mathcal{P}^{(\lambda)}(q) = 0 \qquad (\lambda = 1, 2, 3). \tag{9}$$

Their analysis shows that 1 and 2 modes with polarization vectors

$$\varepsilon_\alpha^{(1)}(q) = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}. \tag{10}$$

are only physical ones in the case under consideration, just as it is in the pure magnetic field [2]. However, it should be emphasized that this coincidence is approximate to within $O(1/\beta)$ and $O(\alpha^2)$ accuracy.

[2]Symbols 1 and 2 correspond to the $\parallel$ and $\perp$ polarizations in pure magnetic field [18] and E- and O- modes in magnetized plasma [6].

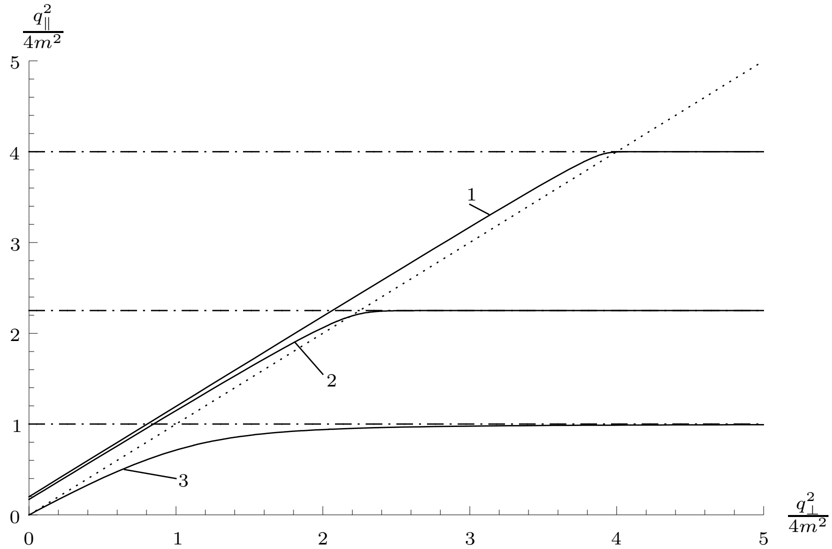

Figure 1: Photon dispersion in a strong magnetic field ($B/B_e = 200$) and degenerate plasma vs. chemical potential $\mu = 1$ MeV – 1, $\mu = 0.75$ MeV – 2 and without plasma – 3. Dotted line corresponds to the vacuum dispersion law, $q^2 = 0$. The angle between the photon momentum and the magnetic field direction is $\pi/2$.

Notice, that in plasma only the eigenvalue $\mathcal{P}^{(2)}(q)$ is modified in comparison with pure magnetic field case. It means that the dispersion law of the mode 1 is the same one as in the magnetized vacuum, where its deviation from the vacuum law, $q^2 = 0$, is negligibly small. From the other hand, the dispersion properties of the mode 2 essentially differ from the magnetized vacuum ones. In the Fig. 1 the photon dispersion in both strong magnetic field and magnetized degenerate plasma are depicted at chemical potential. One can see that in the presence of the magnetized plasma there exist the kinematical region, where $q^2 > 0$ (at $q_\parallel^2 < 4m^2$) contrary to the case of pure magnetic field. This fact could lead to the modification of the kinematics of the different photon-neutrino processes. For example, the photon conversion $\gamma \to \nu\bar{\nu}$ forbidden in the magnetic field without plasma becomes allowed in this region [14]. It is connected with the appearance of the plasma frequency $\omega_{pl}^2 = 2\alpha eBv_F/\pi$.

Moreover, as can be seen from the Fig. 1 in degenerate plasma there is to be the shift of the e^+e^- pair-creation threshold which in pure magnetic field is defined by the relation $q_\parallel^2 = 4m^2$. One can see that in the region $|v_\phi| > 1/v_F$ ($|q_z| < 2p_F$) the last terms in (6) and (8) cancel each other and the only contribution to the imaginary part of $\mathcal{P}^{(2)}(q)$ comes from the logarithm function in (8). It is the fact that leads to the shift of the pair creation threshold from $q_\parallel^2 = 4m^2$ to

$$q_\parallel^2 = 2\left(\mu^2 - p_F|q_z| + \mu\sqrt{(p_F - |q_z|)^2 + m^2}\right). \tag{11}$$

This result is in agreement with simple kinematical analysis of the process $\gamma_2 \to e^+e^-$ in degenerate plasma. Indeed, using the energy and momentum conservation laws with obviuos conditions $E \geqslant \mu$ and $|p_z| \geqslant p_F$ for the electron we come to the result (11).

3 Neutrino emissivity

Our main goal is to obtain the neutrino emissivity in various neutrino reactions. A general expression for neutrino emissivity can be defined in the following way:

$$Q = \frac{1}{V} \int \prod_i \mathrm{d}\Gamma_i f_i \prod_f \mathrm{d}\Gamma_f (1 \pm f_f)\, q_0 \,\frac{|S_{if}|^2}{\tau}, \tag{12}$$

where $\mathrm{d}\Gamma_i$ ($\mathrm{d}\Gamma_f$) are the number of states of initial (final) particles; f_i (f_f) are the corresponding of distribution functions, the sign $+$ $(-)$ corresponds to final bosons (fermions); q_0 is the neutrino pair energy; V is the plasma volume, τ is the interaction time, S_{if} is the S - matrix element.

We will consider the case of relatively low momentum transfers $|q^2| \ll m_W^2$ under calculation of the S - matrix elements. Under this condition, the weak interaction of neutrinos with electrons can be considered in the local limit by using the effective Lagrangian

$$\mathcal{L} \;=\; \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\alpha (C_V + C_A \gamma_5) e\right] j_\alpha \,, \tag{13}$$

where $C_V = \pm 1/2 + 2\sin^2\theta_W$, $C_A = \pm 1/2$, $j_\alpha = \bar{\nu}\gamma_\alpha(1+\gamma_5)\nu$ – is the neutrino current.

The analysis shows, that the integral over phase space in (12) gains its value in the vicinity of the plasma frequency. In this region of the dispersion law for a mode-2 photon can be written as $\omega^2 = q_\perp^2 + q_z^2 + \omega_{pl}^2$. Using the approximation of dispersion law we have obtained the simple expressions for neutrino emissivity of the processes $e\gamma \to e\nu\bar{\nu}$, $\gamma \to \nu\bar{\nu}$ and $\gamma\gamma \to \nu\bar{\nu}$ in the two cases of nonrelativistic and relativistic plasma.

The emissivity due to the photoneutrino process is

$$Q_{\gamma e \to e\nu\bar{\nu}} \simeq \; 1.3 \times 10^{19} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \frac{B}{B_e} \left(\frac{T}{m}\right)^8 \frac{T}{p_F}, \quad \mu \sim m; \tag{14}$$

$$Q_{\gamma e \to e\nu\bar{\nu}} \simeq 5.4 \times 10^{17} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \frac{B}{B_e} \left(\frac{\mu}{m}\right)^5 \left(\frac{T}{\omega_{pl}}\right)^{3/2} \left(\frac{\omega_{pl}}{2\mu} + 1\right) \times$$
$$\times \int_0^1 dx\, (1-x) \frac{(\omega_{pl}/2\mu)^2 - x^2}{1 - \exp\left[-\frac{\mu}{T}(\omega_{pl}/2\mu - x)\right]}, \quad \mu \gg m. \tag{15}$$

The emissivity due to the photon conversion process is

$$Q_{\gamma \to \nu\bar{\nu}} \simeq 10^{21} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \left(\frac{T}{m}\right)^9 \left(\frac{\omega_{pl}}{T}\right)^4 \left[18.7 + 3.3 \left(\frac{\omega_{pl}}{T}\right)^2\right], \quad \mu \sim m; \tag{16}$$

$$Q_{\gamma \to \nu\bar{\nu}} \simeq 10^{20} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \left(\frac{T}{m}\right)^9 \left(\frac{\omega_{pl}}{T}\right)^{15/2} \left[5.5 + 9.0 \frac{T}{\omega_{pl}}\right] \exp\left(-\frac{\omega_{pl}}{T}\right), \quad \mu \gg m. \tag{17}$$

The emissivity due to the two photon annihilation process is

$$Q_{\gamma\gamma \to \nu\bar{\nu}} \simeq 5.3 \times 10^{19} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \left(\frac{\omega_{pl}}{T}\right)^4 \left(\frac{T}{m}\right)^{11}, \quad \mu \sim m\,, \tag{18}$$

$$Q_{\gamma\gamma \to \nu\bar{\nu}} \;\simeq\; 10^{16} \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}} \left(\frac{B}{B_e}\right)^2 \left(\frac{T}{m}\right)^7 \left(\frac{\omega_{pl}}{T}\right)^3 \left(\frac{m}{\mu}\right)^6 \times$$
$$\times \left[2.5 + 2.0 \left(\frac{T}{\omega_{pl}}\right)\right] \exp\left(-\frac{2\omega_{pl}}{T}\right), \quad \mu \gg m\,. \tag{19}$$

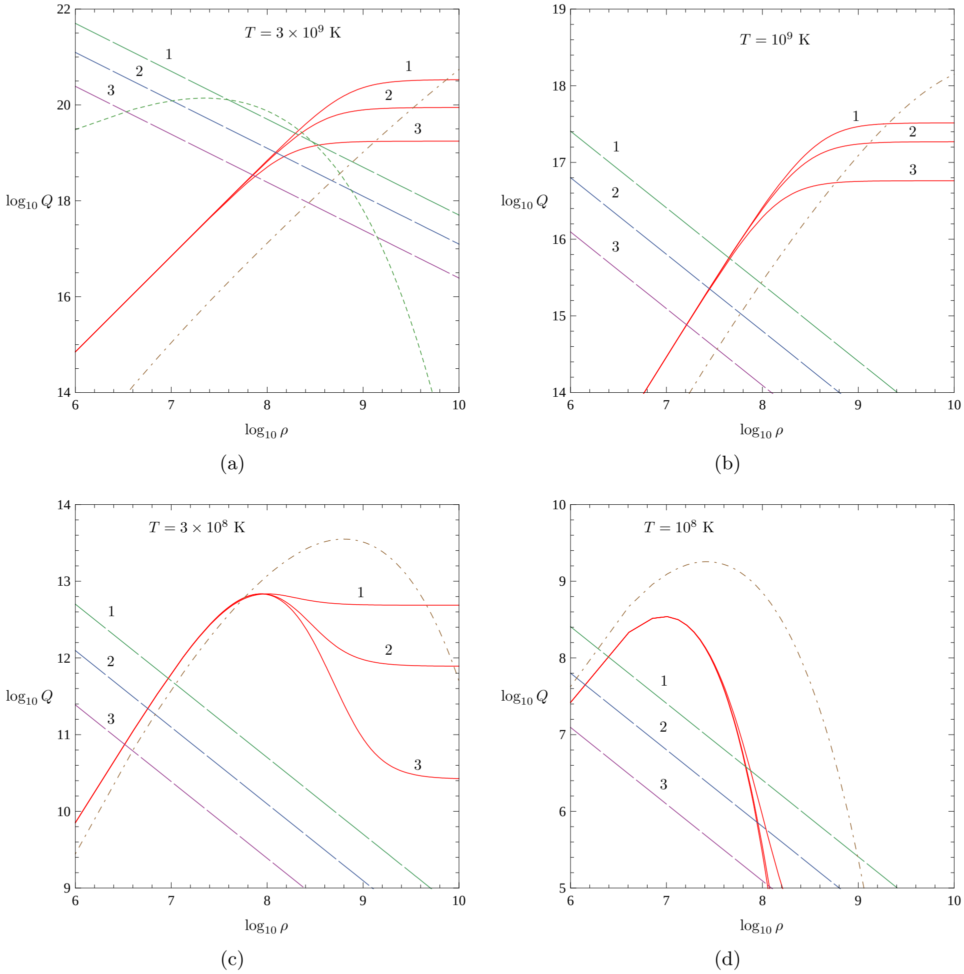

Figure 2: The dependence of the contributions in the neutrino emissivity (erg·cm^{-3} ·s^{-1}) on matter density (g·cm^{-3}) in the outer crust of magnetized neutron star for various temperatures $T = 3 \times 10^9$ K (a), $T = 10^9$ K (b), $T = 3 \times 10^8$ K (c), $T = 10^8$ K (d) and magnetic field values 1 – 10^{16} G, 2 – 5×10^{15} G, 3 – 2.2×10^{15} G. The solid line corresponds to the $\gamma \to \nu\bar{\nu}$ process. The dashed line corresponds to the photoneutrino process. The dotted line corresponds to the e^+e^- pair annihilation process at $B = 0$ [9]. The chain line corresponds to the plasmon decay process [9]. Pair annihilation process become negligible at $T \lesssim 10^9$ K.

We analyse the $\gamma\gamma \to \nu\bar{\nu}$ process contribution in the neutrino emissivity in the regions of the temperature ($10^8 \lesssim T \lesssim 3 \times 10^9$ K), the density ($10^6 \lesssim \rho \lesssim 10^{10}$ g/cm^3) and the magnetic field strength ($B \lesssim 10^{16}$ G.). The obtaining results show, that the influence of this process on the emissivity is suppressed as compared with the contributions of photoneutrino and photon conversion processes under these conditions. Therefore, the possible influence of process $\gamma\gamma \to \nu\bar{\nu}$ on the magnetar cooling is negligible.

4 Application to magnetar cooling

We have made the numerical calculation of the neutrino emissivity dependence on the density caused by processes $\gamma e \to e\nu\bar{\nu}$ (dashed line) and $\gamma \to \nu\bar{\nu}$ (solid line). The results are represented in Figures 2 (a – d) for different temperatures $T = 3 \times 10^9$, 10^9, 3×10^8, 10^8 K and magnetic field strength a – 10^{16} G., b – 5×10^{15} G., c – 2.2×10^{15} G. On the Figures also the e^+e^- pair annihilation process (Fig. 2a) and plasmon decay process (Fig. 2 (a – d)) are shown.

As can be seen from Fig. 2 (a – c) the photoneutrino process is provided the leading contribution in the neutrino emissivity in the density region $10^6 \lesssim \rho \lesssim 10^8$ g/cm^3 at the temperature $T = 3 \times 10^9$, 10^9 K and in the density region $10^6 \lesssim \rho \lesssim 10^7$ g/cm^3 at the temperature $T = 3 \times 10^8$ K. The photon conversion process are suppressed by plasma frequency in this region. On the other hand, it is provided the main contribution in the neutrino emissivity in the density region $10^8 \lesssim \rho \lesssim 10^{10}$ g/cm^3. It is worth noting that the our results are unsuitable for the density $\rho \gtrsim 10^{10}$ g/cm^3.

As the application, we will consider the neutron stars cooling model [9]. The neutrino processes in the neutron stars crust are provided by the neutrino cooling during $10^{-2} \lesssim t \lesssim 100$ years in this model. In addition, the plasmon decay in a weakly magnetized plasma is dominant in this period. On the other hand (see Fig. 2a and 2b), at the magnetic field strength $B \gtrsim 5 \times 10^{15}$ G and the temperature $10^9 \lesssim T \lesssim 3 \times 10^9$ K both processes $\gamma e \to e\nu\bar{\nu}$ and $\gamma \to \nu\bar{\nu}$ are leading as compared with the plasmon decay in a weakly magnetized plasma. Let us discuss the possible consequences of our results.

- Assumig, that the temperature profile in the outer crust weakly depends on magnetic field strength.
- Let us assume also that the magnetar cooling regime at the time $t \gtrsim 10^3$ years is the same one as for the ordinary neutron stars.

Then we can obtain the upper limit for magnetic field value 5×10^{15} G. However, it is rough estimation of magnetic field.

In conclusion, we consider the influence of a strongly magnetized plasma on the photon-neutrino processes $\gamma e^{\pm} \to e^{\pm}\nu\bar{\nu}$, $\gamma \to \nu\bar{\nu}$ and $\gamma\gamma \to \nu\bar{\nu}$. The changes of the photon dispersion properties in a magnetized medium are investigated. We have obtained the simple expressions for neutrino emissivity in the cold plasma limit. These results can be used for the simulation of the magnetar cooling. It is shown, that the possible influence of process $\gamma\gamma \to \nu\bar{\nu}$ on the magnetar cooling is negligible in the regions of temperature ($10^8 \lesssim T \lesssim 3 \times 10^9$ K), density ($10^6 \lesssim \rho \lesssim 10^{10}$ g/cm^3) and magnetic field strength ($B \lesssim 10^{16}$ G.). From the possible modification of the magnetar cooling scenario we have obtained the upper bound on the magnetic field strength $B \lesssim 5 \times 10^{15}$ G.

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